Paper Reference(s)

6681/01

Edexcel GCE

Mechanics M5

Advanced Level

Friday 26 June 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M5), the paper reference (6681), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may gain no credit.

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1. [*In this question* $\mathbf{i}$ *and* $\mathbf{j}$ *are horizontal unit vectors.*]

A small bead of mass 0.5 kg is threaded on a smooth horizontal wire. The bead is initially at rest at the point with position vector $(\mathbf{i} - 6\mathbf{j})$ m. A constant horizontal force $\mathbf{P}$ N then acts on the bead causing it to move along the wire. The bead passes through the point with position vector $(7\mathbf{i} - 14\mathbf{j})$ m with speed $2\sqrt{7}$ m s^{-1}.

Given that $\mathbf{P}$ is parallel to $(6\mathbf{i} + \mathbf{j})$, find $\mathbf{P}$.

(6)

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2. The velocity $\mathbf{v}$ m s^{-1} of a particle P at time t seconds satisfies the vector differential equation

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} + 4\mathbf{v} = \mathbf{0}.$$

The position vector of P at time t seconds is $\mathbf{r}$ metres.

Given that at $t = 0$, $\mathbf{r} = (\mathbf{i} - \mathbf{j})$ and $\mathbf{v} = (-8\mathbf{i} + 4\mathbf{j})$, find $\mathbf{r}$ at time t seconds.

(7)

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3. A system of forces consists of two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting on a rigid body.

$\mathbf{F}_1 = (-2\mathbf{i} + \mathbf{j} - \mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_1 = (\mathbf{i} - \mathbf{j} + \mathbf{k})$ m.

$\mathbf{F}_2 = (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ N and acts at the point with position vector $\mathbf{r}_2 = (4\mathbf{i} - \mathbf{j} - 2\mathbf{k})$ m.

Given that the system is equivalent to a single force $\mathbf{R}$ N, acting at the point with position vector $(5\mathbf{i} + \mathbf{j} - \mathbf{k})$ m, together with a couple $\mathbf{G}$ N m, find

(*a*) $\mathbf{R}$,

(2)

(*b*) the magnitude of $\mathbf{G}$.

(9)

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4. At time $t = 0$ a rocket is launched from rest vertically upwards. The rocket propels itself upwards by expelling burnt fuel vertically downwards with constant speed U m s^{-1} relative to the rocket. The initial mass of the rocket is M_0 kg. At time t seconds, where $t < 2$, its mass is $M_0(1 - \frac{1}{2}t)$ kg, and it is moving upwards with speed v m s^{-1}.

(*a*) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{U}{(2-t)} - 9.8.$$

(7)

(*b*) Hence show that $U > 19.6$.

(2)

(*c*) Find, in terms of U, the speed of the rocket one second after its launch.

(5)

5.

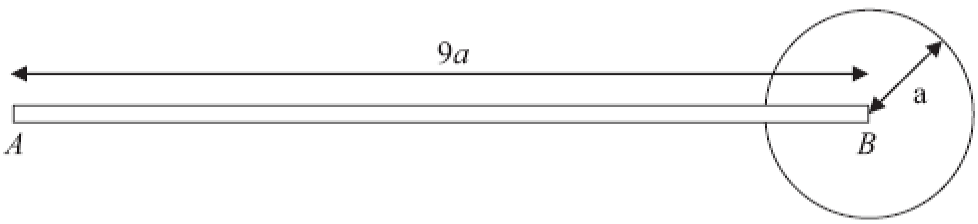

Figure 1

A pendulum P is modelled as a uniform rod AB, of length $9a$ and mass m, rigidly fixed to a uniform circular disc of radius a and mass $2m$. The end B of the rod is attached to the centre of the disc, and the rod lies in the plane of the disc, as shown in Figure 1. The pendulum is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through the end A and is perpendicular to the plane of the disc.

(*a*) Show that the moment of inertia of P about L is $190ma^2$.

(4)

The pendulum makes small oscillations about L.

(*b*) By writing down an equation of motion for P, find the approximate period of these small oscillations.

(7)

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6. A uniform solid right circular cylinder has mass M, height h and radius a. Find, using integration, its moment of inertia about a diameter of one of its circular ends.

[*You may assume without proof that the moment of inertia of a uniform circular disc, of mass m and radius a, about a diameter is* $\frac{1}{4}ma^2$.]

(10)

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7. A uniform square lamina *ABCD*, of mass $2m$ and side $3a\sqrt{2}$, is free to rotate in a vertical plane about a fixed smooth horizontal axis L which passes through A and is perpendicular to the plane of the lamina. The moment of inertia of the lamina about L is $24ma^2$.

The lamina is at rest with C vertically above A. At time $t = 0$ the lamina is slightly displaced. At time t the lamina has rotated through an angle θ.

(*a*) Show that

$$2a\left(\frac{d\theta}{dt}\right)^2 = g(1 - \cos\theta).$$

(4)

(*b*) Show that, at time t, the magnitude of the component of the force acting on the lamina at A, in a direction perpendicular to AC, is $\frac{1}{2}mg\sin\theta$.

(7)

When the lamina reaches the position with C vertically below A, it receives an impulse which acts at C, in the plane of the lamina and in a direction which is perpendicular to the line AC. As a result of this impulse the lamina is brought immediately to rest.

(*c*) Find the magnitude of the impulse.

(5)

TOTAL FOR PAPER: 75 MARKS

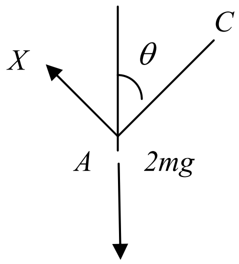
END

June 2008
6681 Mechanics M5
Mark Scheme

Question Number	Scheme	Marks
1.	$\mathbf{d} = (7\mathbf{i} - 14\mathbf{j}) - (\mathbf{i} - 6\mathbf{j}) = (6\mathbf{i} - 8\mathbf{j})$	B1
	$(6k\mathbf{i} + k\mathbf{j}).(6\mathbf{i} - 8\mathbf{j}) = \frac{1}{2} \text{x} \frac{1}{2} \text{x } (2\sqrt{7})^2$	M1 A2 ft
	$28k = 7 \Rightarrow k = \frac{1}{4}$	D M1
	$\Rightarrow \mathbf{P} = \frac{\mathbf{3}}{\mathbf{2}}\mathbf{i} + \frac{1}{4}\mathbf{j}$	A1 **6**
2.	Aux Equn: $m^2 + 4m = 0 \Rightarrow m = 0 \text{ or } -4$	M1
	$\mathbf{r} = \mathbf{A} + \mathbf{B}\mathrm{e}^{-4t}$	
	$t = 0, \mathbf{r} = \mathbf{i} - \mathbf{j}$: $\quad \mathbf{A} + \mathbf{B} = \mathbf{i} - \mathbf{j}$	A1
	$\mathbf{v} = -\mathbf{4B}\mathrm{e}^{-4t}$	M1
	$t = 0, v = -8\mathbf{i} + 4\mathbf{j}$: $\quad -\mathbf{4B} = -8\mathbf{i} + 4\mathbf{j}$	M1
	$\mathbf{B} = 2\mathbf{i} - \mathbf{j} \Rightarrow \mathbf{A} = -\mathbf{i}$	
	so, $\mathbf{r} = -\mathbf{i} + (2\mathbf{i} - \mathbf{j})\mathrm{e}^{-4t}$	A1 A1
	$= (2\mathrm{e}^{-4t} - 1)\mathbf{i} - \mathrm{e}^{-4t}\mathbf{j}$	A1 **7**
3.(a)	$\mathbf{R} = (-2\mathbf{i} + \mathbf{j} - \mathbf{k}) + (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$	M1
	$= (\mathbf{i} + \mathbf{k})$	A1 (2)
(b)	$\mathbf{G} + (5\mathbf{i} + \mathbf{j} - \mathbf{k}) \text{ x } (\mathbf{i} + \mathbf{k}) = (\mathbf{i} - \mathbf{j} + \mathbf{k}) \text{ x } (-2\mathbf{i} + \mathbf{j} - \mathbf{k}) + (4\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \text{ x } (3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$	M1 A2 ft
	$\mathbf{G} + (\mathbf{i} - 6\mathbf{j} - \mathbf{k}) = (-\mathbf{j} - \mathbf{k}) + (-4\mathbf{i} - 14\mathbf{j} - \mathbf{k})$	A3 ft
	$\mathbf{G} = (-5\mathbf{i} - 9\mathbf{j} - \mathbf{k})$	A1
	$\lvert\mathbf{G}\rvert = \sqrt{(-5^2 + (-9)^2 + (-1)^2} = \sqrt{107}$ Nm	M1 A1 (9)
		11

4. (a)	$-mg\,\delta t = (m+\delta m)(v+\delta v) + \delta m(U-v) - mv$ $-mg\,\delta t = mv + m\,\delta v + v\,\delta m + U\,\delta m - v\,\delta m - mv$ $-mg = m\dfrac{\mathrm{d}v}{\mathrm{d}t} + U\dfrac{\mathrm{d}m}{\mathrm{d}t}$ $m = M_0(1-\tfrac{1}{2}t) \Rightarrow \dfrac{\mathrm{d}m}{\mathrm{d}t} = -\tfrac{1}{2}M_0$ $-M_0 g(1-\tfrac{1}{2}t) = M_0(1-\tfrac{1}{2}t)\dfrac{\mathrm{d}v}{\mathrm{d}t} - \tfrac{1}{2}M_0 U$ $U - g(2-t) = (2-t)\dfrac{\mathrm{d}v}{\mathrm{d}t}$ $\dfrac{U}{(2-t)} - 9.8 = \dfrac{\mathrm{d}v}{\mathrm{d}t}$ *	M1 A2 A1 B1 M1 A1 (7)
(b)	$\dfrac{\mathrm{d}v}{\mathrm{d}t} > 0$ when $t = 0 \Rightarrow \dfrac{U}{2} - 9.8 > 0$ $\Rightarrow \quad U > 19.6$ *	M1 A1 (2)
(c)	$v = \int \dfrac{U}{(2-t)} - 9.8\ \mathrm{d}t$ $= -U\ln(2-t) - 9.8t + C$ $t = 0, v = 0:\ 0 = -U\ln 2 + C \Rightarrow C = U\ln 2$ so, $v = U\ln\dfrac{2}{(2-t)} - 9.8t$ $t = 1:\ v = U\ln 2 - 9.8$	M1 A1 M1 M1 A1 (5) **14**

Question Number	Scheme	Marks
5.(a)	$I = \frac{1}{3}m(9a)^2 + \frac{1}{2}2ma^2 + 2m(9a)^2$	M1 A1 A1
	$= 27ma^2 + ma^2 + 162ma^2$	
	$= 190ma^2$	A1* (4)
(b)	M(L),	
	$mg\frac{9a}{2}\sin\theta + mg9a\sin\theta = -190ma^2\ddot{\theta}$	M1 A2
	$\ddot{\theta} = -\frac{9g}{76a}\sin\theta$	
	For small θ, $\sin\theta \approx \theta$,	M1
	$\Rightarrow \ddot{\theta} = -\frac{9g}{76a}\theta$ so S.H.M.	A1
	Period $= 2\pi\sqrt{\frac{76a}{9g}} = \frac{4\pi}{3}\sqrt{\frac{19a}{g}}$	DM1 A1
		(7)
		11
6.	$\delta m = \pi a^2 \delta x . \frac{M}{\pi a^2 h} = \frac{M\delta x}{h}$	M1 A1
	$\delta I = \frac{1}{4}\delta m . a^2 + \delta m . x^2$	M1 A1
	$= \frac{M}{4h}(a^2 + 4x^2)\delta x$	M1 A1
	$I = \int_0^h \frac{M}{4h}(a^2 + 4x^2)\mathrm{d}x$	M1 A1
	$= \frac{M}{4h}\left[a^2 x + \frac{4}{3}x^3\right]_0^h$	M1
	$= \frac{M}{4}(a^2 + \frac{4}{3}h^2)$	
	$= \frac{M}{12}(3a^2 + 4h^2)$	A1
		10

7.(a)		
	$\frac{1}{2}24ma^2\dot{\theta}^2 = 2mg.3a(1-\cos\theta)$	M1 A1 A1
	$2a\dot{\theta}^2 = g(1-\cos\theta)$	A1 (4)
(b)	$(\searrow)$	
	$2mg\sin\theta - X = 2m3a\cos\theta$	M1 A2
	$\mathrm{M}(L),\quad 2mg.3a\sin\theta = 24ma^2\ddot{\theta}$	M1 A1
	$\Rightarrow X = 2mg\sin\theta - 6ma\left(\frac{g\sin\theta}{4a}\right)$	DM1
	$= \frac{mg\sin\theta}{2}*$	A1 (7)
(c)	$\theta = \pi : 2a\dot{\theta}^2 = g(1-\cos\pi)$	M1
	$\dot{\theta} = \sqrt{\frac{g}{a}}$	A1
	$6a.I = 24ma^2\sqrt{\frac{g}{a}}$	M1 A1
	$\Rightarrow I = 4m\sqrt{ag}$	A1
		(5)
		16